## REVISED SIMPLEX METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEM.

**ADELEKE KABIR ODUNAYO MATRICULATION NUMBER: 20172521**

### DEPARTMENT OF MATHEMATICS COLLEGE OF PHYSICAL SCIENCES

**FEDERAL UNIVERSITY OF AGRICULTURE, ABEOKUTA**

**IN PARTIAL FULFILMENT OF THE AWARD OF BACHELOR OF SCIENCE DEGREE IN MATHEMATICS**

**FEBRUARY, 2023**

## DECLARATION

I hereby declare that this research was written by me and is a correct record of my own research. It has not been presented in any previous application for any degree of this or any other University. All citations and sources of information are clearly acknowledged by means of references.

#### ADELEKE KABIR ODUNAYO Date:. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .

## CERTIFICATION

This is to certify that this research work entitled **REVISED SIMPLEX METHOD FOR SOLVING LINEAR PROGRAMMING PROBLEM** is the outcome of the research work carried out by ADELEKE KABIR ODUNAYO (20172521) in the Depart- ment of Mathematics, Federal University of Agriculture, Abeokuta, Ogun State.

*. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .*

#### PROF. O.A. OSINUGA Date

(**SUPERVISOR**)

*. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . .*

#### DR. E.O. ADELEKE Date

(**Ag. HEAD OF DEPARTMENT**)

## DEDICATION

This project work is dedicated to Almighty God, the creator of the universe and all mankind, who gave me this grace from the inception of this project work till its completion. And also to my wonderful family, starting from my beloved parents, Mr and Mrs Adeleke as well as my ever-supportive siblings and to everyone that has been supportive and helpful in my education life.

## ACKNOWLEDGMENTS

All glory, honour and adoration is to the Almighty God who has made the success of the research work and the completion of my BSc. programme at large a reality.

I would like to express my gratitude and appreciation to my supervisor, Prof I.A Osinuga whose help in stimulating suggestion and encouragement helped in the process of complet- ing this project. I also sincerely thank him for the time spent proofreading and correcting my many mistakes.

I am grateful to the the Head of Department, DR. E.O. Adeleke , immediate past Head of department Prof. B.I. Olajuwon and all lecturers of the Department of Mathematics, because all I have been taught from my first year in the Department made it possible for me to carry out this research work.

My sincere appreciation also goes to my parent Mr and Mrs Adeleke for their full support, advice, prayer, love and care placed on me throughout this project period and my stay on campus at large. Daddy and Mummy, I pray to God that you live long to eat the fruit of your labor.

My profound appreciation also goes to my wonderful siblings for their immensive contri- bution physically, spiritually, financially towards the success of my programme. I pray that almighty God take them to higher grounds.

Finally, my sincere gratitude also goes to all those who have contributed to my success in FUNAAB: my friends, departmental mates and many others that i couldnt mention their names.Thank you all and God bless you. (AMEN).

## ABSTRACT

**Table of Contents**

[Declaration](#_bookmark0) i

[Certification](#_bookmark1) ii

[Dedication](#_bookmark1) iii

[Acknowledgements](#_bookmark1) iv

[Abstract](#_bookmark1) v

|  |  |  |  |
| --- | --- | --- | --- |
| [**1 INTRODUCTION**](#_bookmark2)  [1.1 Background to the Study](#_bookmark3) . . . . . . . . . . . . . . . . . . . . . . . . . . | | .1 | **1** |
| [1.2 Motivation](#_bookmark4) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | | .3 |  |
| [1.3 Objectives](#_bookmark5) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | | .3 |  |
| [1.4 Defintion of Terms](#_bookmark6) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | | .3 |  |
| [**2 LITERATURE REVIEW**](#_bookmark7)  [2.1 Introduction](#_bookmark8) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | | .6 | **6** |
| [2.2 Concept of Linear Programming](#_bookmark9) . . . . . . . . . . . . . . . . . . . . . . | | .7 |  |
| [2.3 Components of Linear Programming](#_bookmark10) . . . . . . . . . . . . . . . . . . . | | .7 |  |
| [2.4 Characteristics of Linear Programming](#_bookmark11) . . . . . . . . . . . . . . . . . . | | .9 |  |
| [2.5 Linear Programming Problems](#_bookmark12) . . . . . . . . . . . . . . . . . . . . . . . | | .10 |  |
| [2.6 Linear Programming Applications](#_bookmark13) . . . . . . . . . . . . . . . . . . . . . | | .10 |  |
| [2.7 Importance of Linear Programming](#_bookmark14) . . . . . . . . . . . . . . . . . . . . | | .11 |  |
| [**3**](#_bookmark15) | [**METHODOLOGY**](#_bookmark15)  [3.1 Methods to Solve Linear Programming Problems](#_bookmark16) . . . . . . . . . . . . | .12 | **12** |
|  | [3.1.1 Linear Programming Simplex Method](#_bookmark17) . . . . . . . . . . . . . . . | .12 |  |

[3.1.2 Graphical Method](#_bookmark18) 13

#### [APPLICATIONS](#_bookmark19) 14

* 1. [Illustrative examples](#_bookmark20) 14

|  |  |  |
| --- | --- | --- |
| [**5 CONCLUSION AND RECOMMENDATIONS**](#_bookmark21) |  | **26** |
| [5.1 Conclusion](#_bookmark22) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | .26 |  |
| [5.2 Recommendations](#_bookmark23) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | .27 |  |
| [References](#_bookmark24) . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | .28 |  |

**Chapter 1 INTRODUCTION**

* 1. **Background to the Study**

The revised simplex method is technically equivalent to the traditional simplex method, but it is implemented differently. It retains a representation of a basis of the matrix containing the constraints, rather than a tableau that directly depicts the constraints scaled to a set of fundamental variables.

When using the regular simplex approach to solve a linear programming problem on a digital computer, the full simplex table must be stored in the computer table’s memory, which may not be possible for particularly big problems. However, each iteration must include the calculation of each table. The revised simplex method, which is a variation of the original approach, uses fewer computer resources since it computes and maintains only the data that is currently needed for testing and/or improving the current solution. To put it another way, it only requires a small amount of effort. i.e

* + - The non-basic variable that reaches the basis is determined using the net evaluation row ∆*j*. The pivoting column
    - To establish the minimal positive ratio, first, identify the present basis variables and their values (*XB* column), and then identify the basis variable to exit the basis.

By using the inverse of the current basis matrix at any iteration, the above information

can be directly extracted from the original equations.

For the revised simplex method, there are two standard versions.

**Standard form-I** – In this form, an identity matrix is considered to be obtained after just adding slack variables.

**Standard form-II** – When artificial variables are required for an identity matrix, the two-phase approach of the ordinary simplex method is applied in a slightly different manner.

Revised simplex method steps are as follows:

**Step 1:** Formalize the problem in standard form – 1

* + - Confirm that all *bi ≥* 0.
    - Maximization should be the objective function.
    - Inequalities are converted to equations using non-negative slack variables.
    - The first constraint equation is also treated as the objective function.

**Step 2:** In the revised simplex form, build the starting table. Using appropriate notation, express the result of step 1 as a matrix.

**Step 3:** For *a*1 and *a*2, Compute ∆*j*. **Step 4:** Conduct an optimality test. **Step 5:** Determine the *Xk* column vector.

**Step 6:** Find the outgoing vector. We’re not supposed to do any calculations for the Z-row.

**Step 7:** Choose a better solution.

Linear programming on the other hand is a method of optimising operations with some constraints. The main objective of linear programming is to maximize or minimize the numerical value. It consists of linear functions which are subjected to the constraints in the form of linear equations or in the form of inequalities. Linear programming is considered an important technique that is used to find the optimum resource utilisation. The term “linear programming” consists of two words as linear and programming. The word “linear” defines the relationship between multiple variables with degree one. The word “programming”

defines the process of selecting the best solution from various alternatives.

Linear Programming is widely used in Mathematics and some other fields such as economics, business, telecommunication, and manufacturing fields. In this article, let us discuss the definition of linear programming, its components, and different methods to solve linear programming problems.

## Motivation

This Research work was motivated by the variation in the result gotten when solving linear programming problem with Simplex Method and Revised Simplex Method

## Objectives

The main objective of this research is to solve linear programming problem using the revised simplex method.

Specifically, the study seeks:

iTo review some methods of solving Linear Programming Problem iiSolve linear programming problems using the revised simplex method

## Defintion of Terms

**Definition 1.4.1.Simplex method:** is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem.

**Definition 1.4.2.Simplex tableau:** is used to perform row operations on the linear programming model as well as for checking optimality.

**Definition 1.4.3.Slack variables:** are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints.

**Definition 1.4.4.Standard form:** is the baseline format for all linear programs before solving for the optimal solution.

**Definition 1.4.5.Standard form of a linear programming problem:** A linear programming problem is said to be a standard maximization problem in standard form if its mathematical model is of the following form:

Max *P* = *c*1*x*1 + *c*2*x*2 + *· · · cnxn* Subject to *a*11*x*1 + *a*12*x*2 + *· · ·* + *a*1*nxn ≤ b*1

*· · ·*

*am*1*x*1 + *am*2*x*2 + *· · ·* + *amn ≤ bm x*1*, x*2*, · · · , xn ≥* 0

where *x*1; *x*2; *· · ·* ; *xn* are decision variables, *c*1; *· · ·* ; *cn, a*11 *· · ·* ; *amn* are any real numbers, and *b*1*, · · · , bm ≥* 0 are nonnegative real numbers.

**Note:** Any linear programming problem (in the form we defined earlier) can be converted into the standard maximization problem in standard form.

**Definition 1.4.6.Basic Solution:** Given an LP with n decision variables and m constraints, a basic solution of the corresponding initial system is a solution of the initial systems (not taking into account nonnegative constraints) in which n of the variables *x*1*, · · · xn, s*1*, · · · sm* are equal to zero.

**Note:** the list of variables *x*1*, · · · xn, s*1*, · · · , sm, n* of which should be zero, does not contain P.

**Definition 1.4.7.Basic Feasible Solution** : If a basic solution of the initial system corresponds to a certain point in the feasible region of the original LP, then it is called a basic feasible solution.

**Theorem 1.4.1. *Fundamental Theorem of Linear Pro-gramming:*** *If the optimal value of the objective function in a linear programming problem exists, then that value must occur at one or moreof the basic feasible solutions of the initial system.*

**Definition 1.4.8.Basic and Nonbasic Variables:** The variables of a basic solution that are assumed to be zero arecalled nonbasic variables. All the remaining variables are calledbasic variables.

**Definition 1.4.9.Constraints:** are a series of equalities and inequalities that are a set of criteria necessary to satisfywhen finding the optimal solution.

**Definition 1.4.10.Inequality:** is an expression that does not have one definite solution and is distinguishable by its ‘greater than’ or ‘less than’ symbols in the place of a traditional equal sign.

**Definition 1.4.11.Linear program:** is a model used to achieve the best outcome given a maximum or minimum equation with linear constraints.

#### Definition 1.4.12.Entering and Exiting Variables:

A non basic variable that is chosen to become a basic variableat a particular step of the simplex method is called entering variable.

A basic variable that is chosen to become a nonbasic variable at aparticular step of the simplex method is called exiting variable.

**Definition 1.4.13.Pivot Column:** The column corresponding to the entering variable is called thepivot column.

**Definition 1.4.14.Pivot Row and Pivot Element:** The row corresponding to the exiting variable is called the pivot row.

The element at the intersection of the pivot column and the pivotrow is called the pivot element

**Definition 1.4.15.Optimal solution:** of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model.

# Chapter 2 LITERATURE REVIEW

## Introduction

Simplex method also called simplex technique or simplex algorithm was developed by

G.B. Dantzig, an American mathematician. Simplex method is suitable for solving linear programming problems with a large number of variables. The method through an iterative process progressively approaches and ultimately reaches to the maximum or minimum values of the objective function. The Simplex method is an approach to solving linear programming models by hand using slack variables, tableaus, and pivot variables as a means to finding the optimal solution of an optimization problem. A linear program is a method of achieving the best outcome given a maximum or minimum equation with linear constraints. Most linear programs can be solved using an online solver such as MatLab, but the Simplex method is a technique for solving linear programs by hand. To solve a linear programming model using the Simplex method the following steps are necessary:

* + Standard form
  + Introducing slack variables
  + Creating the tableau
  + Pivot variables
  + Creating a new tableau
  + Checking for optimality
  + Identify optimal values

## Concept of Linear Programming

Linear programming (LP) or Linear Optimisation may be defined as the problem of maximizing or minimizing a linear function that is subjected to linear constraints. The constraints may be equalities or inequalities. The optimisation problems involve the calculation of profit and loss. Linear programming problems are an important class of optimisation problems, that helps to find the feasible region and optimise the solution in order to have the highest or lowest value of the function.

In other words, linear programming is considered as an optimization method to maximize or minimize the objective function of the given mathematical model with the set of some requirements which are represented in the linear relationship. The main aim of the linear programming problem is to find the optimal solution.

Linear programming is the method of considering different inequalities relevant to a situation and calculating the best value that is required to be obtained in those conditions. Some of the assumptions taken while working with linear programming are:

* + - The number of constraints should be expressed in the quantitative terms
    - The relationship between the constraints and the objective function should be linear
    - The linear function (i.e., objective function) is to be optimised

## Components of Linear Programming

The following are the elements , parts , or basic components of linear programming model

* + - Decision variables
    - Objective function
    - Constraints
    - Non - negativity
    - Data

1. **Decision variables:** Decision variables are the quantities that are to be calculated. They are the variables in a mathematical programming model that are unknown. Decision variables are physical quantities that the decision - maker has control over. The nature of the objective function and the availability of resources guide the evaluation of various alternatives and selection of the best to arrive at the optimal value of the objective function. Certain activities ( also known as decision variables ) are carried out for this purpose, which are typically denoted by *X*1*, X*2*, · · · Xn*. The value of these variables indicates the extent to which each of these is executed. For example, in a product - mix production situation, an LP model, may be used to determine units of each item to be created by using limited resources like money, material, machine etc.

The decision-maker may or may not be able to control the value of certain variables. Variables are called controllable if their values are under the control of the decision- maker, otherwise, they are called uncontrollable. These decision variables, which are frequently interrelated in terms of resource usage, necessitate simultaneous solutions. All decision variables in an LP model are continuous, controllable, and non-negative, that is, *x*1 *≥* 0*, X*2 *≥* 0*, · · · , xn ≥* 0*.*

1. **Objective function:** The objective function depicts how each decision variable would influence the cost, or, merely, the value that requires to be optimised. The graphical or simplex methods are used to find the optimal value of given objective function.

In other words, each LPP’s objective functions is expressed in term of decision variable to optimize the criterion of optimality (also known as measure of performance

), such as revenue, expenses (costs), profits, time, distance, etc.

An objective function in its general form is represented as:

Optimize (Maximize or Minimize )*Z* = *c*1*x*1 + *c*2*x*2 + *· · · , cnxn*

where,

* + Z represents the measure-of-performance variable, which is a function of

*x*1*, x*2*, x*3*, · · · xn* and

* + *c*1*, c*2*, c*3*, · · · , cn* are parameters that represents the contribution of a unit of the corresponding variables *x*1*, x*2*, · · · , xn* to the measure of performance.

1. **Constraints:** Constraints represent how each decision variable would make use of limited quantities of resources. Constraints are physical, financial, legal, ethical, technological, or other limitations, on what numerical values can be assigned to the decision variables.

We know that there are always limitations on the use of resources, such as manpower, machine, raw material, storage, money, and so on, which limit the extent to which an objective can be achieved. Such constraints must be expressed as linear equalities or inequalities in terms of decision variables. An LP model’s solution must satisfy these constraints.

1. **Non-negativity:** The non-negativity criterion is a vital part of the LP model because the values of decision variables make sense and relate to real-world problems.
2. **Data:** These measures the connections between the objective function and the constraints .

## Characteristics of Linear Programming

The following are the six characteristics of the linear programming problem:

1. **Constraints:** The limitations should be expressed in the mathematical form, regarding the resource.
2. **Objective Function:** In a problem, the objective function should be specified in a quantitative way.
3. **Linearity:** The relationship between two or more variables in the function must be linear. It means that the degree of the variable is one.
4. **Finiteness:** There should be finite and infinite input and output numbers. In case, if the function has infinite factors, the optimal solution is not feasible.
5. **Non-negativity:** The variable value should be positive or zero. It should not be a negative value.
6. **Decision Variables:** The decision variable will decide the output. It gives the ultimate solution of the problem. For any problem, the first step is to identify the decision variables.

## Linear Programming Problems

The Linear Programming Problems (LPP) is a problem that is concerned with finding the optimal value of the given linear function. The optimal value can be either maximum value or minimum value. Here, the given linear function is considered an objective function. The objective function can contain several variables, which are subjected to the conditions and it has to satisfy the set of linear inequalities called linear constraints. The linear programming problems can be used to get the optimal solution for the following scenarios, such as manufacturing problems, diet problems, transportation problems, allocation problems and so on.

## Linear Programming Applications

A real-time example would be considering the limitations of labours and materials and finding the best production levels for maximum profit in particular circumstances. It is

part of a vital area of mathematics known as optimisation techniques. The applications of LP in some other fields are

* + - **Engineering** – It solves design and manufacturing problems as it is helpful for doing shape optimisation
    - **Efficient Manufacturing** – To maximise profit, companies use linear expressions
    - **Energy Industry** – It provides methods to optimise the electric power system.
    - **Transportation Optimisation** – For cost and time efficiency.

## Importance of Linear Programming

Linear programming is broadly applied in the field of optimisation for many reasons. Many functional problems in operations analysis can be represented as linear programming problems. Some special problems of linear programming are such as network flow queries and multi-commodity flow queries are deemed to be important to have produced much research on functional algorithms for their solution.

# Chapter 3 METHODOLOGY

## Methods to Solve Linear Programming Problems

The linear programming problem can be solved using different methods, such as the graphical method, simplex method, or by using tools such as R, open solver etc. Here, we will discuss the two most important techniques called the simplex method and graphical method in detail.

### Linear Programming Simplex Method

The simplex method is one of the most popular methods to solve linear programming problems. It is an iterative process to get the feasible optimal solution. In this method, the value of the basic variable keeps transforming to obtain the maximum value for the objective function. The algorithm for linear programming simplex method is provided below:

* + - * **Step 1:** Establish a given problem. (i.e.,) write the inequality constraints and objective function.
      * **Step 2:** Convert the given inequalities to equations by adding the slack variable to each inequality expression.
      * **Step 3:** Create the initial simplex tableau. Write the objective function at the bottom row. Here, each inequality constraint appears in its own row. Now, we can represent the problem in the form of an augmented matrix, which is called the initial simplex tableau.
      * **Step 4:** Identify the greatest negative entry in the bottom row, which helps to identify the pivot column. The greatest negative entry in the bottom row defines the largest coefficient in the objective function, which will help us to increase the value of the objective function as fastest as possible.
      * **Step 5:** Compute the quotients. To calculate the quotient, we need to divide the entries in the far right column by the entries in the first column, excluding the bottom row. The smallest quotient identifies the row. The row identified in this step and the element identified in the step will be taken as the pivot element.
      * **Step 6:** Carry out pivoting to make all other entries in column is zero.
      * **Step 7:** If there are no negative entries in the bottom row, end the process. Otherwise, start from step 4.
      * **Step 8:** Finally, determine the solution associated with the final simplex tableau.

### Graphical Method

The graphical method is used to optimize the two-variable linear programming. If the problem has two decision variables, a graphical method is the best method to find the optimal solution. In this method, the set of inequalities are subjected to constraints. Then the inequalities are plotted in the XY plane. Once, all the inequalities are plotted in the XY graph, the intersecting region will help to decide the feasible region. The feasible region will provide the optimal solution as well as explains what all values our model can take. Let us see an example here and understand the concept of linear programming in a better way.

# Chapter 4 APPLICATIONS

## 4.1Illustrative examples

This section apply the step by step revised Simplex method to solve a linear programming problem to find the optimal solution.

#### Example 4.1.1.

Minimize: *− z* = *−*8*x*10*x*2 *−* 7*x*3 Subject to: *x*1 + 3*x*2 + 2*x*3 *≤* 10

*− x*1 *−* 5*x*2 *− x*3 *≥ −*8

*x*1*, x*2*, x*3 *≥* 0

#### Solution

**Step 1: Standard Form**

Standard form is the baseline format for all linear programs before solving for the optimal solution and has three requirements:

1. Must be a maximization problem,
2. All linear constraints must be in a less-than-or-equal-to inequality. 3.All variables are non-negative.

These requirements can always be satisfied by transforming any given linear program

using basic algebra and substitution. Standard form is necessary because it creates an

ideal starting point for solving the Simplex method as efficiently as possible as well as other methods of solving optimization problems.

To transform a minimization linear program model into a maximization linear program model, simply multiply both the left and the right sides of the objective function by -1.

*−*1 *×* (*−z* = *−* 8*x*1 + 10*x*2 *−* 7*x*3

=*⇒ z* =8*x*1 + 10*x*2 + 7*x*3 Hence, we have: *z* =8*x*1 + 10*x*2 + 7*x*3

Transforming linear constraints from a greater-than-or-equal-to inequality to a less-than- or-equal-to inequality can be done similarly as what was done to the objective function. By multiplying by -1 on both sides, the inequality can be changed to less-than-or-equal-to.

*−*1*×*(*−x*1 *−* 5*x*2 *− x*3 *≥ −*8)

*x*1 + 5*x*2 + *x*3 *≤* 8

Once the model is in standard form, the slack variables can be added as shown in Step 2 of the Simplex method.

#### Step 2: Determine Slack Variables

Slack variables are additional variables that are introduced into the linear constraints of a linear program to transform them from inequality constraints to equality constraints. If the model is in standard form, the slack variables will always have a +1 coefficient. Slack variables are needed in the constraints to transform them into solvable equalities with one definite answer.

*x*1 + 3*x*2 + 2*x*3 + ***s*1** = 10

*x*1 + 5*x*2 + *x*3 + ***s*2** = 8

*x*1*, x*2*, x*3*,* ***s*3** *≥* 0

After the slack variables are introduced, the tableau can be set up to check for optimality

as described in Step 3.

#### Step 3: Setting up the Tableau

A Simplex tableau is used to perform row operations on the linear programming model as well as to check a solution for optimality. The tableau consists of the coefficient corresponding to the linear constraint variables and the coefficients of the objective function. In the tableau below, the bolded top row of the tableau states what each column represents. The following two rows represent the linear constraint variable coefficients from the linear programming model, and the last row represents the objective function variable coefficients.

maximize: *z* = 8*x*1 + 10*x*2 + 7*x*3 Subject to: *x*1 + 3*x*2 + 2*x*3 + ***s*1** = 10

*x*1 + 5*x*2 + *x*3 + ***s*2** = 8

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 *x*3 | *s*1 | *s*2 | *z* | *b* |
| 1 | 3 2 | 1 | 0 | 0 | 10 |
| 1 | 5 1 | 0 | 1 | 0 | 8 |
| -8 | -10 -7 | 0 | 0 | 1 | 0 |

Table 4.1: Coefficient corresponding to the linear constraint variables and the coefficients of the objective function

Once the tableau has been completed, the model can be checked for an optimal solution as shown in Step 4.

#### Step 4: Check Optimality

The optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. The optimal solution would exist on the corner points of the graph of the entire model. To check optimality using the tableau, all values in the last row must contain values greater than or equal to zero. If a value is less than zero, it means that variable has not reached its optimal value. As seen in the previous tableau, three negative values exists in the bottom row indicating that this solution is not optimal. If a tableau is not optimal, the next step is to identify the pivot variable to base a new tableau on, as described in Step 5.

#### Step 5: Identify Pivot Variable

The pivot variable is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by looking at the bottom row of the tableau and the indicator. Assuming that the solution is not optimal, pick the smallest negative value in the bottom row. One of the values lying in the column of this value will be the pivot variable. To find the indicator, divide the beta values of the linear constraints by their corresponding values from the column containing the possible pivot variable. The intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row will become the pivot variable.

In the example shown below, *−*10 is the smallest negative in the last row. This will

designate the *x*2 column to contain the pivot variable. Solving for the indicator gives us a

value of 10 for the first constraint, and a value of 8 for the second constraint. Due to 8

3

5

5

being the smallest non-negative indicator, the pivot value will be in the second row and have a value of 5.

Indicator

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 | *x*3 | *s*1 | *s*2 | *z* | *b* |
| 1 | 3 | 2 | 1 | 0 | 0 | 10 |
| 1 | 5 | 1 | 0 | 1 | 0 | 8 |
| -8 | -10 | -7 | 0 | 0 | 1 | 0 |

10

3



8

5

Table 4.2

Now that the new pivot variable has been identified, the new tableau can be created in Step 6 to optimize the variable and find the new possible optimal solution.

**Step 6: Create the New Tableau** The new tableau will be used to identify a new possible optimal solution. Now that the pivot variable has been identified in Step 5, row operations can be performed to optimize the pivot variable while keeping the rest of the tableau equivalent.

1. To optimize the pivot variable, it will need to be transformed into a unit value

(value of 1). To transform the value, multiply the row containing the pivot variable

by the reciprocal of the pivot value. In the example below, the pivot variable is originally 5, so multiply the entire row by 1 .

5



*x*1

1

5

*x*2

1

*x*3

1

5

*s*1

0

*s*2

1

5

*z*

0

*b*

8

5

Table 4.3

1. After the unit value has been determined, the other values in the column containing the unit value will become zero. This is because the *x*2 in the second constraint is being optimized, which requires *x*2 in the other equations to be zero.



*x*1

*x*2

0

1

*x*3 *s*1 *s*2 *z b*

1

5

0

1

5

0

8

5

Table 4.4

1. In order to keep the tableau equivalent, the other variables not contained in the pivot column or pivot row must be calculated by using the new pivot values. For each new value, multiply the negative of the value in the old pivot column by the value in the new pivot row that corresponds to the value being calculated. Then add this to the old value from the old tableau to produce the new value for the new tableau. This step can be condensed into the equation on the next page:

New tableau value = (Negative value in old tableau pivot column) *×* (value in new tableau pivot row) + (Old tableau value)

#### Old Tableau:

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 | *x*3 | *s*1 | *s*2 | *z* | *b* |
| 1 | 3 | 2 | 1 | 0 | 0 | 10 |
| 1 | 5 | 1 | 0 | 1 | 0 | 8 |
| -8 | -10 | -7 | 0 | 0 | 1 | 0 |

Table 4.5



#### NewTableau:



*x*1

2

5

-6

1

5

*x*2

0

1

0

*x*3

7

5

-5

1

5

*s*1

1

0

0

*s*2

*−*3

5

1

2

5

*z*

0

0

1

*b*

26

5

16

8

5

Table 4.6

Numerical examples are provided below to help explain this concept a little better.

#### Numerical examples:

1.To find the *s*2 value in row 1:

New tableau value = (Negative value in old tableau pivot column) *×* (value in new tableau pivot row) + (Old tableau value)

This implies that New tableau value = (-3) *×* ( 1 ) + 0 = *−*3

5 5

To find the *x*1 variable in row 3:

New tableau value = (Negative value in old tableau pivot column) *×* (value in new tableau pivot row) + (Old tableau value)

New value = (10) *×* ( 1 ) + (-8) = -6

5

Once the new tableau has been completed, the model can be checked for an optimal solution. **Step 7: Check Optimality**

As explained in Step 4, the optimal solution of a maximization linear programming model are the values assigned to the variables in the objective function to give the largest zeta value. Optimality will need to be checked after each new tableau to see if a new pivot variable needs to be identified. A solution is considered optimal if all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, the solution is considered optimal and Steps 8 through 11 can be ignored. If negative values exist, the solution is still not optimal and a new pivot point will need to be determined which is demonstrated in Step 8.

#### Step 8: Identify New Pivot Variable

If the solution has been identified as not optimal, a new pivot variable will need to be determined. The pivot variable was introduced in Step 5 and is used in row operations to identify which variable will become the unit value and is a key factor in the conversion of the unit value. The pivot variable can be identified by the intersection of the row with the smallest non-negative indicator and the smallest negative value in the bottom row.

Indicator

*x*1

2

5

*−*1

5

-6

*x*2

0

1

0

*x*3

7

5

1

-5

*s*1

1

0

0

*s*2

*−*3

5

1

5

2

*z*

0

0

1

*b*

26

5

8

5

0

26 */* 2 = 13

5 5

8 */* 1 = 8

5 5

Table 4.7

With the new pivot variable identified, the new tableau can be created in Step 9.

#### Step 9: Create New Tableau

After the new pivot variable has been identified, a new tableau will need to be created. Introduced in Step 6, the tableau is used to optimize the pivot variable while keeping the rest of the tableau equivalent.

1. Make the pivot variable 1 by multiplying the row containing the pivot variable by the reciprocal of the pivot value. In the tableau below, the pivot value was 1 , so

5

everything is multiplied by 5.



*x*1 *x*2 *x*3 *s*1 *s*2 *z b*

1 5 1 0 1 0 8

Table 4.8

1. Next, make the other values in the column of the pivot variable zero. This is done by taking the negative of the old value in the pivot column and multiplying it by

the new value in the pivot row. That value is then added to the old value that is being replaced.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 | *x*3 | *s*1 | *s*2 | *z* | *b* |
| 0 | -2 | 1 | 1 | -1 | 0 | 2 |
| 1 | 5 | 1 | 0 | 1 | 0 | 8 |
| 0 | 30 | 1 | 0 | 8 | 1 | 64 |

Table 4.9



#### Step 10: Check Optimality

Using the new tableau, check for optimality. Explained in Step 4, an optimal solution appears when all values in the bottom row are greater than or equal to zero. If all values are greater than or equal to zero, skip to Step 12 because optimality has been reached. If negative values still exist, repeat steps 8 and 9 until an optimal solution is obtained.

#### Step 11: Identify Optimal Values

Once the tableau is proven optimal the optimal values can be identified. These can be found by distinguishing the basic and non-basic variables. A basic variable can be classified to have a single 1 value in its column and the rest be all zeros. If a variable does not meet this criteria, it is considered non-basic. If a variable is non-basic it means the optimal solution of that variable is zero. If a variable is basic, the row that contains the 1 value will correspond to the beta value. The beta value will represent the optimal solution for the given variable.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| *x*1 | *x*2 | *x*3 | *s*1 | *s*2 | *z* | *b* |
| 0 | -2 | 1 | 1 | -1 | 0 | 2 |
| 1 | 5 | 1 | 0 | 1 | 0 | 8 |
| 0 | 30 | 1 | 0 | 8 | 1 | 64 |

Table 4.10



Basic variables: *x*1*, s*1*, z*

Non-basic variables: *x*2*, x*3*, s*2

For the variable *x*1, the 1 is found in the second row. This shows that the optimal *x*1

value is found in the second row of the beta values, which is 8.

Variable *s*1 has a 1 value in the first row, showing the optimal value to be 2 from the beta column. Due to *s*1 being a slack variable, it is not actually included in the optimal solution since the variable is not contained in the objective function.

The zeta variable has a 1 in the last row. This shows that the maximum objective value will be 64 from the beta column.

The final solution shows each of the variables having values of:

*x*1 = 8 *s*1 = 2

*x*2 = 0 *s*2 = 0

*x*3 = 0 *z* = 64

The maximum optimal value is 64 and found at (8,0,0) of the objective function.

#### Example 4.1.2.

Subsector:

*MaxZ* = 6*x*1 *−* 2*x*2 + 3*x*3 (4.1)

2*x*1 *− x*2 + 2*x*3 *≤* 2

*x*1 + 4*x*2 *≤* 4

*x*1*, x*2*, x*3 *≥* 0

#### Solution

**Step 1:** Convert the LPP into standard form adding slack variable *s*1 and (*s*2)

*max Z* = 6*x*1 *−* 2*x*2 + 3*x*2 + 0*s*1 + 0*s*2

2*x*1 *− x*2 + 2*x*3 + *s*1 = 2

*x*1 + 4*x*2 + *s*2 = 4

*x*1*, x*2*, x*3*, s*1*, s*2 *≥* 0

The Initial Basic Feasible Solution

**Step 2:** Find the The Initial Basic Feasible Solution with initial basic *B* = *I*0

*x*1 *x*2 *x*3 *s*1 *s*2 *A* = 2 *−*1 2 1 0

 

 

 1 0 4 0 1 

*b* = 2 *c* = .6 *−*2 3 0 0Σ

*x*1 *x*2 *x*3 *s*1 *s*2 *A*ˆ =  *A*  =  2 *−*1 2 1 0 

 4

   

 

2

*b* =   = 4

 *b*

*−C*

1 0 4 0 1

 6 *−*2 3 0 0 

0

0

*B* = .*s s* Σ = 1 0 = *B−*1

1 2 0 1

*C* = .

Σ = *C*

*B−*1 = .

Σ 1 0 = . Σ

*B* 0 0 *B* 0 0

 *B* 1 0

*−*

0 1 0 0

1 0 0

*B*ˆ*−*1 = 

 = 0 1 0

*CBB−*1 1  

0 0 1

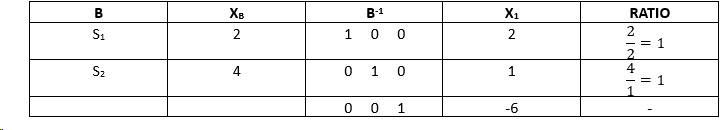


Figure 4.1

2 *−*1 2 1 0

.  Σ . Σ

*Zj − Cj* = (*CBB−*1)*A*ˆ = 0 0 1 = 1 0 4 0 1 = 6 2 3 0 0

 *−*

6 *−*2 3 0 0

*Z*1 *− C*1 = 6 most negative so *x*1 enters the basis Compute *X*ˆ*k* = *B*ˆ*−*1 *− a*ˆ*k*

1 0 0

 2 

*X*ˆ1 = *B*ˆ*−*1 *− a*ˆ*k* = 0 1 0 =  1 

0 0 1

*−*6

   

1 0 0 2 2

*X*ˆ*B* = *B*ˆ*−*1ˆ*b* = 0 1 0 4 = 4

Find the minimum of Σ *X*ˆ*B , X*

*X*

*>* 0Σ

0 0 1 0 0

*k*

*k*

*S*1 leaves the basis, so 2 is the key element

 1 0 0 *R*1





2

2

1 *R*1

1 *− −− >* 2

*R*

*R*

2

1

*B*ˆ*−*1 *−* ˆ*b* = *−* 1

1 0 *R*2

1 *− −− > R*2 *− R*1

*R*

2

1

 3 0 1 *R*3

1 *− −− > R*3 + *R*1

2 *−*1 2 1 0

. Σ   . Σ

*Zj − Cj* = (*CBB−*1)*A*ˆ = 3 0 1 = 1 0 4 0 1 = 0 1 3 3 0

  *−*

6 *−*2 3 0 0

*Z*2 *− C*2 = *−*1, most negative,

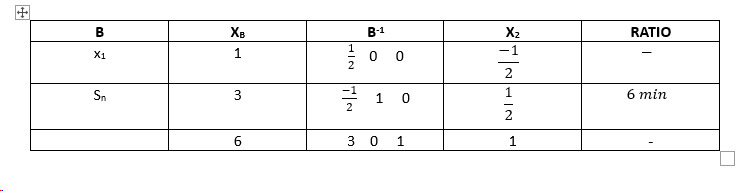


Figure 4.2

Therefore, *x*2 enters the basis

 1

2

0 0 *−*1

*−* 1 

2

ˆ      

*X*2 = *B*ˆ*−*1*a*ˆ2 = *−* 1

2

1 0  0  =  1 

     

2

3 0 1  2 

*−*1

 1 0 0 2



2

2

1

*X*ˆ*B* = *B*ˆ*−*1ˆ*b* = *−* 1 1 0 4 = 3

     

*s*2 leaves the basis 1

2

3 0 1 0 6

is the key element

 0 1 0

*B*ˆ*−*1 = *−*1 2 0

 2 2 1

*R*2 *− −− > R*2*x*2 *R*1 *− −− > R*1 + *R*2 *R*3 *− −− > R*3 + *R*2

2 *−*1 2 1 0

2

*Zj − Cj* = (*CBB−*1)*A*ˆ = .2 2 1Σ = 1 0 4 0 1 = .0 0 9 2 2Σ

 *−*

6 *−*2 3 0 0 Since all *Zj − Cj ≥* 0, the current feasible solution is optimal

 0 1 0 2

*X*ˆ*B* = *B*ˆ*−*1ˆ*b* = *−*1 2 0 4 =  6  *x*2

 4  *x*1

 2 2 1 0

12 *z*

The Optimal Solution is therefore: *x*1 = 4 *x*2 = 6 *x*3 = 0 *z* = 12

# Chapter 5

**CONCLUSION AND RECOMMENDATIONS**

## Conclusion

The Simplex method is an approach for determining the optimal value of a linear program by hand. The method produces an optimal solution to satisfy the given constraints and produce a maximum zeta value. To use the Simplex method, a given linear programming model needs to be in standard form, where slack variables can then be introduced. Using the tableau and pivot variables, an optimal solution can be reached. From the example worked throughout, it can be determined that the optimal objective value is 64 and can be found when *x*1 = 8*, x*2 = 0*, andx*3 = 0.

## Recommendations

I Recommend that other researchers should also apply other methods simpler than this to solve linear programming problems.

# References

1. M.S. Bazaraa, J.J. Jarvis, H.D. Sherali, (1990). Linear Programming and Network Flows.
2. https://byjus.com/maths/graphical-method-linear-programming/
3. h[ttps://www.studocu.com/ro](http://www.studocu.com/row/document/mount-kenya-university/research-)w/doc[ument/mount-k](http://www.studocu.com/row/document/mount-kenya-university/research-)en[ya-university/research-](http://www.studocu.com/row/document/mount-kenya-university/research-) proposal-writing-skills/ch06-1-2-simplex-method/14181330
4. https://prinsli.com/components-of-linear-programming/